The University of Alabama at Birmingham (UAB)
Department of Physics

PH 461/561 – Classical Mechanics I – Fall 2005

Assignment # 7 Due: **Thursday, October 13**
(Turn in for credit!)

Activities in preparation for Oct. 13 lecture:

1. Consider a cart on a spring which is critically damped.

   a. Assuming that at time $t = 0$ the cart is sitting at its equilibrium position and is kicked in the positive direction with velocity v_0, find its position $x(t)$ for all subsequent times and sketch a graph of your answer.

   b. Do the same for the case when the cart is released from rest at position $x = x_0$. In this case, how far is the cart from equilibrium after a time equal to $T_0 = 2\pi/\omega_0$, the period in the absence of any damping?

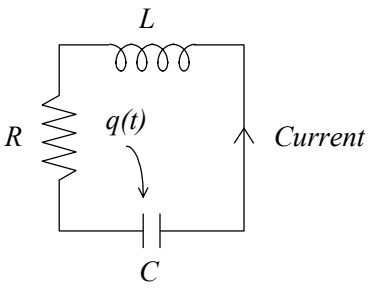
2. An undamped oscillator has period $T_0 = 1.000\,\text{s}$. A little damping is added to this oscillator so that its period changes to $T_d = 1.001\,\text{s}$
   a. What is the damping factor γ?
   b. By what factor will the amplitude of oscillation decrease after 10 cycles?
   c. Which effect of damping would be more noticeable, the change of period or the decrease of the amplitude?

3. Find the Q factor for the oscillator of Problem 2 above.

4. The charge $q(t)$ on the capacitor of the *RLC* circuit below obeys the differential equation

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = 0$$

Find the Q factor for this *RLC* circuit if $R = 2\ \Omega$; $C = 10^{-13}$ Farad; $L = 10^{-3}$ Henry.



5. The position $x(t)$ of an overdamped oscillator is given by

$$x(t) = C_1 e^{-\left(\gamma - \sqrt{\gamma^2 - \omega_0^2}\right)t} + C_2 e^{-\left(\gamma + \sqrt{\gamma^2 - \omega_0^2}\right)t}$$

   a. Find the constants C_1 and C_2 in terms of the initial position x_0 and initial velocity v_0.

   b. Sketch the behavior of $x(t)$ for the two cases:
      i. $v_0 = 0$ and $x_0 \neq 0$
      ii. $v_0 \neq 0$ and $x_0 = 0$

   c. Show that if you let $\gamma \to 0$, your solution $x(t)$ in part (a) approaches the correct solution for undamped motion.